# A Potential Flow Design Method for Multicomponent Airfoil Sections

J.L. Kennedy* and D.J. Marsden†
*University of Alberta, Edmonton, Alberta, Canada*

**A multicomponent, potential flow design method has been developed to generate an airfoil section with a specified velocity distribution on its surface. The method was developed from an accurate and efficient surface vorticity analysis method. An iterative approach is used to adjust the geometry of a basic airfoil section until it gives the required velocity distribution. This approach allows the designer to adjust the solution to suit the practical constraints within which airfoil designers must work. Examples of the results of various airfoil design problems are shown.**

## Nomenclature

$C$ = airfoil chord, also control point
$C_L$ = lift coefficient
$K$ = influence coefficient (matrix element)
$M$ = number of components in airfoil section
$N$ = number of surface elements on a component
$r$ = distance defined in Fig. 1
$U$ = velocity on airfoil surface
$U_\infty$ = freestream velocity
$x,y$ = Cartesian coordinates
$\alpha$ = angle of attack
$\gamma$ = vortex density
$\psi$ = stream function

*Subscripts*

$i$ = index of control point
$j$ = index of surface element
$k$ = index of airfoil component
$tp$ = trailing control point

*Superscripts*

$r$ = required value
$d$ = design iteration number

## Introduction

As the requirements of modern airfoil sections become more demanding, there is a need for accurate analytical methods for use by design engineers. Potential flow analysis methods will, for a given geometry of section, give the designer the surface velocity distribution around the section. By using this, the lift, pitching moment, and boundary-layer characteristics of the section can be calculated. Combining experience with sound aerodynamic principles, the designer can develop a surface velocity distribution which will give a desired set of characteristics. An accurate and efficient technique is needed to calculate the airfoil section shape which will produce this required velocity distribution.

To date, the only practical, analytical methods of multicomponent airfoil section design are based on surface singularity techniques. While some exact transformation methods have been developed for single component sections, no working methods are known for the multicomponent case. Surface singularity methods are also desirable because, as the design process is iterative, adjustments can be made to both the designed geometry and the required surface velocities at each iteration. This allows the designer to include many of the constraints which frequently occur in the design of modern airfoil sections.

Surface vortex analysis methods can always be converted to give a design technique. A desired surface velocity is specified, which is equivalent to specifying the strength of the singularity in such a method. This cannot be done for surface source methods and for this reason they are not employed directly in the design process.

The first use of a surface vortex design method is that of Goldstein and Jerison[1] who developed a technique of airfoil and cascade design. This technique sought to find the location of the vortices of known strength, such that each lay on the same streamline. This streamline is then the airfoil section. The properties of this section were determined by a conformal mapping technique since surface singularity analysis techniques were not then available. With the advent of such analysis methods, Wilkinson[2] developed a technique in which the suction side velocities are specified and the camber of a specified thickness section is adjusted to satisfy the requirements. The convergence on the required solution is checked at each iteration by the analysis method. Two major limitations with this technique are that the velocities can be specified on only one side of the section and the thickness distribution of the section must be known in advance.

Both Chen[3] and Mavriplis[4] have developed similar design techniques based on improved analysis methods. These methods do not have the limitations that are inherent in Wilkinson's technique and are consequently much more powerful. In these methods the design process follows naturally from the analysis method and the equations of the design method are developed from those found in the analysis. Recently Beatty and Narramore[5] have combined an accurate surface source analysis method with Wilkinson's design technique in order to achieve better accuracy. The method, however, still contains the major limitations inherent in Wilkinson's method.

The design method presented here is basically the same as those developed by Chen[3] and Mavriplis,[4] but it has been developed from the more accurate and efficient analysis method presented by the authors in a previous paper.[6] Accuracy and efficiency are particularly important because the design process is iterative. The major difference between the analysis method of Ref. 6 and earlier techniques was the use of a trailing point Kutta condition which reduced the number of control points required for an accurate solution to about one third of those required by previous methods.

This design method has a great deal of flexibility and some examples are shown to illustrate its application and convergence characteristics.

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*Graduate Student, Department of Mechanical Engineering.

†Professor, Department of Mechanical Engineering. Associate Fellow C.A.S.I.

## The Design Equations

The surface singularity analysis method presented by the authors in Ref. 6 made use of a vorticity density $\gamma(S')$, distributed on the airfoil surface. The boundary condition used to solve for the vorticity distribution required that the surface of each component of the airfoil should be a streamline of the resulting flow. Using the notation of Fig. 1, the stream function $\psi_k$ of the $k$th component can be written as:

$$\psi_k = y_S\cos\alpha - x_S\sin\alpha - \frac{1}{2\pi}\int_{S'}\gamma(S')\ln r(S,S')\,\mathrm{d}S' \qquad (1)$$

In order to solve for $\gamma(S')$ and $\psi_k$, the integral in Eq. (1) is replaced by a summation over a set of $N$ straight-line surface elements, $S_j$. The midpoint of each of these, $x_j$, $y_j$, is termed a control point $C_j$, and each element has constant vorticity of strength $\gamma_j$ distributed along its length.

Applying Eq. (1) at the $N$ control points yields the system of equations

$$\psi_k + \sum_{j=1}^{N} K_{ij}\gamma_j = y_i\cos\alpha - x_i\sin\alpha \qquad (i=1,2,3,\ldots N) \qquad (2)$$

where $K_{ij}$ is the influence coefficient of the $j$th element on control point $i$. To solve for the $N$ values of $\gamma$, and the values of $\psi_k$, we need $M$ other equations in addition to the $N$ equations represented by Eq. (2). The airfoil Kutta condition provides these additional equations. An additional control point $C_{tp}$, is chosen just downstream of the trailing edge located on the bisector of the trailing edge angle at a distance of 0.005 of the length of the trailing edge element. The resulting equation is

$$\psi_k + \sum_{j=1}^{N} K_{tpj},\gamma_j = y_{tp}\cos\alpha - x_{tp}\sin\alpha \qquad (3)$$

In the case of a multicomponent section there is one such Kutta condition for each component.

Values of $x_i$, $y_i$, and $K_{ij}$ are given by the airfoil geometry and Eqs. (2) and (3) are solved for the surface velocity distribution, $\gamma_j$. However, for the airfoil design problem, values of $\gamma_j$ are given and the equations are to be solved for the required section geometry, $x_i$, $y_i$.

A direct solution to yield the desired geometry does not appear to be possible using Eqs. (2) and (3), but these equations can be used in an iterative procedure in which the geometry of a basic airfoil section is gradually modified until it gives the desired velocity distribution.

Substituting the required values $\psi^{(r)}$ and $\gamma_j^{(r)}$ in Eqs. (2) and (3) while retaining the $K_{ij}$ from the previous design allows one to solve for the geometry of a modified section. Generally, the $x_i$, $x_{tp}$ values can be kept constant and new values of $y_i$, $y_{tp}$ calculated.

The values of $\psi^{(r)}$ chosen is somewhat arbitrary since it only has the effect of moving the airfoil up or down relative to the $x,y$ coordinate system without changing any other characteristics of the flow. In the case of multicomponent wing sections, the increment in stream function from one component to the next is important because it represents the flow through the slot between the components. Choice of values of $\psi$ is discussed in more detail below in connection with the example for design of a two-component airfoil.

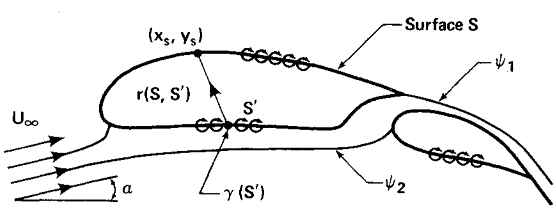


**Fig. 1 Vortex representation of a two-component airfoil.**

At the $d$th iteration of the design procedure one has

$$y_i^{(d)} = \frac{1}{\cos\alpha}x_i\sin\alpha + \psi_k^{(r)} + \sum_{j=1}^{N} K_{ij}^{(d-1)}\gamma^{(r)} \qquad (i=1,2\ldots N) \qquad (4)$$

for the control points, while the trailing points are calculated from

$$y_{tp}^{(d)} = \frac{1}{\cos\alpha}x_{tp}\sin\alpha + \psi_k^{(r)} + \sum_{j=1}^{N} K_{tp,j}^{(d-1)}\gamma_j^{(r)} \qquad (5)$$

The location of the trailing point is so close to the trailing edge that it is assumed that $y_{tp}^{(d)}$ is the designed location of the trailing edge.

To start the design process an arbitrary basic airfoil section is used to generate the initial set of influence coefficients $K_{ij}^{(0)}$. Thereafter the $K_{ij}$ for each new iteration is calculated for the coordinates obtained from the previous iteration. At each iteration the coefficient matrix can be used to calculate the actual velocities on the designed section for comparison with the required velocities. The process is repeated until a satisfactory design is achieved.

The design angle of attack $\alpha$ is the angle between the freestream direction and the $x$ axis. Consequently the choice of $\alpha$ determines the orientation of the final design and for single component sections this will mainly be along the $x$ axis or at small angles to it. For example, choosing $\alpha = 0$ deg and a velocity distribution $\gamma_j^{(r)}$ corresponding to, say, a $C_L$ of 1, the designed section chord may be inclined at an angle of about 10 deg to the $x$ axis. If $\alpha = 10$ deg had been chosen, the designed section would then have been aligned approximately with the $x$ axis.

In the technique previously described, the control points of the section are adjusted from iteration to iteration along the vertical lines $x_i$ = constant. For wing sections with components whose individual chords are approximately vertical, for example, flaps at large deflections, this may not be suitable since in these cases it is the length and not the thickness which would be adjusted. The adjustment to the geometry could be made in other directions. However, a limitation of this method is that the adjustment cannot be made along lines parallel to the freestream direction.

Whether this technique will converge on a solution will depend largely on the manner in which the influence coefficients vary between iterations. The elements of the coefficient matrix $K_{ij}$ are generally small, the size depending on both the airfoil section and the number of elements $N$ being used. In general, the diagonal terms (the self-influence coefficients) are the largest. These terms are functions only of the length of the elements being considered. In progressing from design to design the lengths of elements should change only very slowly. Hence the terms which are generally the largest in the summation in Eq. (4) change slowly and this acts to stabilize the design procedure.

## Determination of Airfoil Coordinates

The coordinates $y_i$ that are obtained from Eq. (4) are the element control points or midpoints, while element endpoints are needed to calculate the $K_{ij}$ for the next iteration. The endpoints could be determined by passing a curve through the designed midpoints and interpolating. However, in subsequent calculations the control point is taken to be the midpoint of the straight line joining the endpoints, resulting in the midpoint's moving inwards as shown in Fig. 2a.

The error will be largest near the leading edge where the radius of curvature is the smallest. This will result in poor accuracy in cases where the angle of attack is large. While

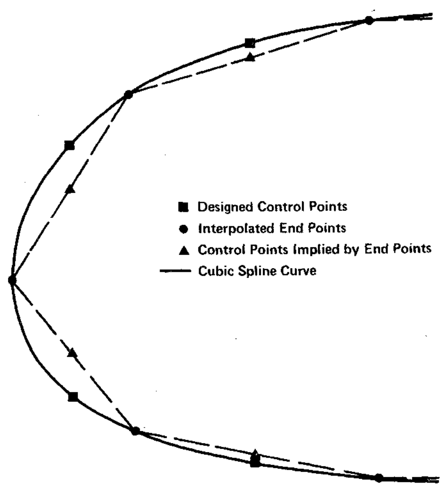


**Fig. 2a Design of nose region when smoothing.**

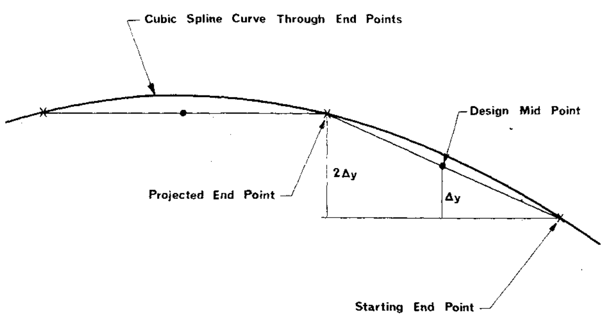


**Fig. 2b Location of endpoints by projection.**

many schemes can be devised to correct for this error, a simple angular rotation of each surface about the trailing edge, which brings the control points at the nose to their designed location, has been adopted here and has been reasonably successful.

Alternatively, if one endpoint can be located correctly, the adjacent endpoint can be located by simply projecting a straight line through the designed midpoint. Since the midpoint is halfway between the two endpoints, the increment in $y$ is just twice the increment between the known endpoint and the midpoint. A sketch is shown in Fig. 2b. The trailing edge point found from the Kutta condition [Eq. (5)] provides a starting point for both upper and lower surfaces. The leading edge point calculated from both surfaces should be the same, and typically the two values are less than 0.05% of chord apart. The leading edge is taken as the mean of these two values. The advantage of this method is that the airfoil will have its control points at exactly the designed location. A disadvantage is that small errors can propagate as the surface is projected, which may result in a saw-toothed surface shape. In such cases the smoothing method can be used to ensure a smooth airfoil shape.

## Procedure

The flow chart shown in Fig. 3 indicates the general design procedure and some of the options. A starting airfoil section is needed to determine the initial values, $K_{ij}$, of the coefficient matrix. The required velocity distribution is then used to calculate the new set of control points from which the designed section geometry is calculated by either the shooting or smoothing methods.

Choice of a suitable required velocity distribution can be a problem in that an arbitrarily chosen distribution may not necessarily correspond to a possible airfoil shape. The rigid requirements of a completely specified velocity distribution can frequently be relaxed in practical cases as, in general, only

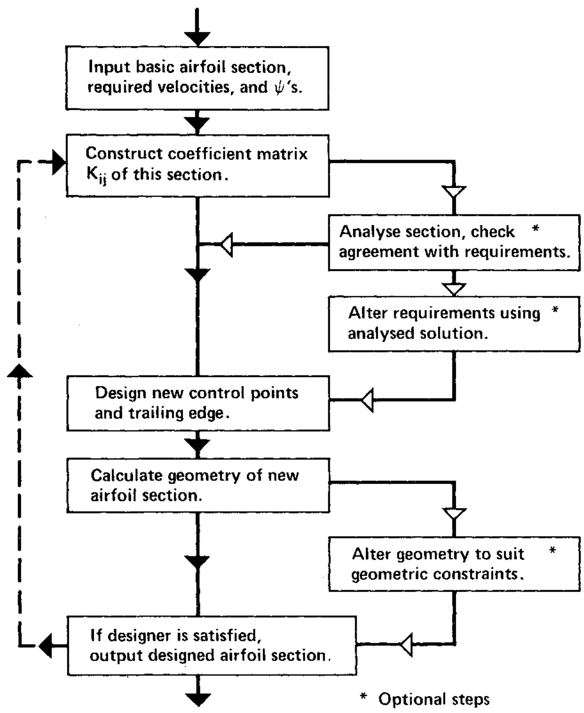


**Fig. 3 Flow chart of solution method.**

one surface is critical at any angle of attack. For example, one may want to specify only the upper surface velocity distribution. In this case, analysis must be performed on the basic section, and the lower surface velocities thus calculated are used as the required lower surface velocities in the design. The analysis is kept in the iteration loop and the continual updating of the lower surface requirements causes the lower surface geometry to change slowly while being compatible with the required upper surface velocities.

Alterations of the designed section shape to suit geometric constraints is a further option shown in Fig. 3. For the previous example, the designed section may turn out to have an undesirable lower surface. The designer could arbitrarily change this lower surface either to a predetermined desirable shape or to maintain a particular thickness distribution on the section (the latter making this method comparable to Wilkinson's[2] technique). This, of course, affects the lower surface velocity distribution and the upper surface distribution to a lesser extent. Further iterations would come closer to the required lower surface geometry and the required upper surface velocity distribution.

Another important geometric constraint that must be included is one that prevents the upper and lower surfaces from crossing. A minimum wedge angle at the trailing edge can be specified that is not too restrictive but will ensure that a practical airfoil section evolves. Here an angle of approximately 1 deg has been used quite successfully.

Further practical constraints are imposed on multicomponent airfoil designs. The flap must retract into the main section to form a good airfoil for cruising flight. This usually means that the main component geometry is fixed and only the shape of the flap and the slot can be designed. Even the flap shape may be constrained to conform to the desired cruise airfoil when retracted. Such constraints are easily included in this method.

## Results

The iterative design procedure should converge on an airfoil design which gives exactly the required velocity distribution. As the technique is numerical and iterative the results will not give exact agreement and it is first necessary to determine just how close this method will come to the required

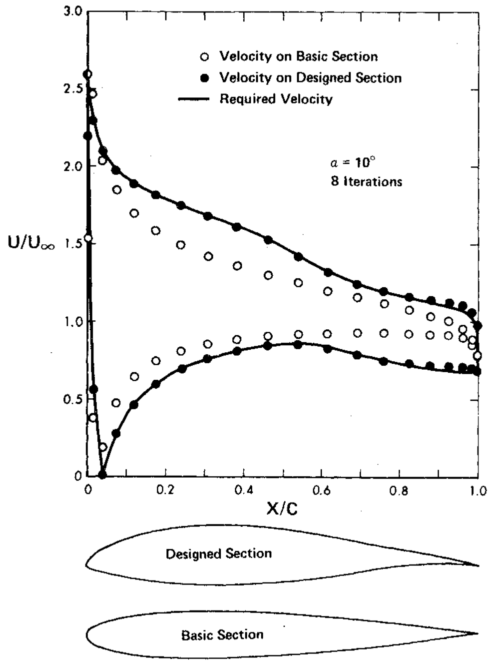


Fig. 4 Single component design.

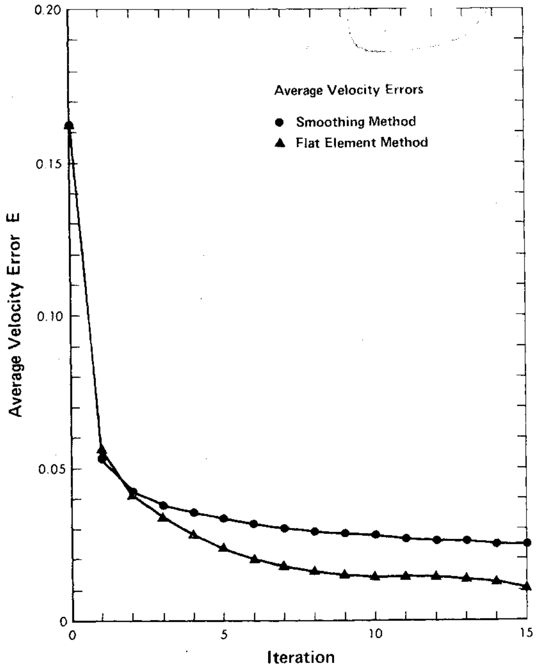


Fig. 6 Convergence of single component design.

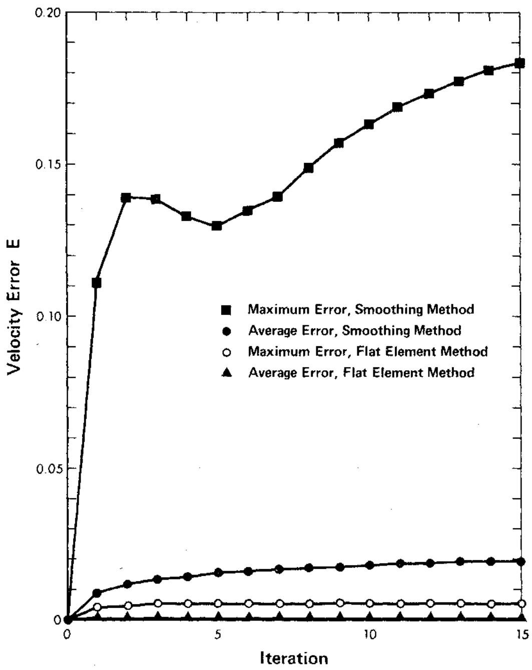


Fig. 5 Error growth.

solution in practical cases. An example of a practical airfoil design is shown in Fig. 4. As can be seen, there is good, but not perfect, agreement between the designed and required velocity distributions. The velocity error at any control point is defined by

$$E = \left| \frac{U - U^{(r)}}{U_\infty} \right| \tag{6}$$

The major effect of the design procedure is to reduce the error as the number of iterations increases. However, the error introduced by the technique of determining the airfoil coordinates from the designed control points tend to grow as the number of iterations increases. This limits the method in its attempt to achieve the required velocity exactly. To some extent this can be examined by setting the required velocities identical to those on the initial basic airfoil. The airfoil should then continually design itself, and in a perfect design technique the velocity errors $E$ should always be zero.

Both techniques of determining the airfoil coordinates, the flat element "shooting" method and the cubic spline smoothing method, were tested. A Wortmann FX-61-163 wing section at $\alpha = 10$ deg was used as a test case. This section has the required velocity distribution shown in Fig. 4. Using 40 surface elements, the maximum and average velocity errors for both techniques are plotted in Fig. 5. From this it is clear that the flat element method is superior. The maximum errors in this method occur near the trailing edge where the approximation that the trailing point is the trailing edge is made. In the smoothing method the maximum errors occur at the nose where the effects of reducing the surface curvature are felt most accutely. The effect of not adjusting the airfoil to match the control points at the nose was a 50% to 100% increase in the errors. Small geometric errors at the nose are greatly aggravated by the high angle of attack of the section; hence this is a severe test of the smoothing technique. The maximum errors, although they appear large, amount to less than 7% of the local velocity at the nose.

As a test of the design method, the FX-61-163 airfoil was designed using its known velocity distribution at $\alpha = 10$ deg and starting with a NACA 0012 airfoil as the basic section. Forty surface elements were used, since the analysis method[6] has shown that this is adequate for highly accurate solutions from single component sections. The design method was applied 15 times, with and without smoothing, to determine the relative rates of convergence. The average velocity errors at each iteration are plotted in Fig. 6. The first few iterations provide the most rapid convergence and the rate of convergence gradually decreases from then on. The smoothing technique provides slower convergence than the flat element technique, as could be expected.

Defining the term improvement as the difference between the average errors for any two adjacent iterations, it is clear that the improvement decreases as more iterations are performed. There is therefore a point at which the design process should be terminated. The criterion used must provide a realistic balance between the need for accurate design and the cost of performing each iteration. In designing single component sections the design process is terminated when the improvement becomes less than 1% of the original average

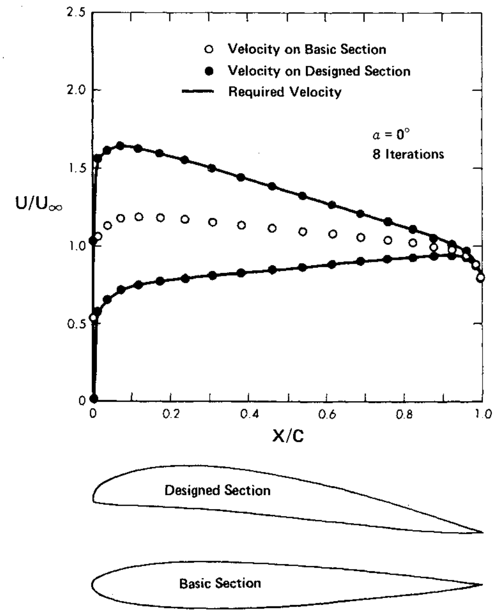


**Fig. 7 Arbitrary single component design.**

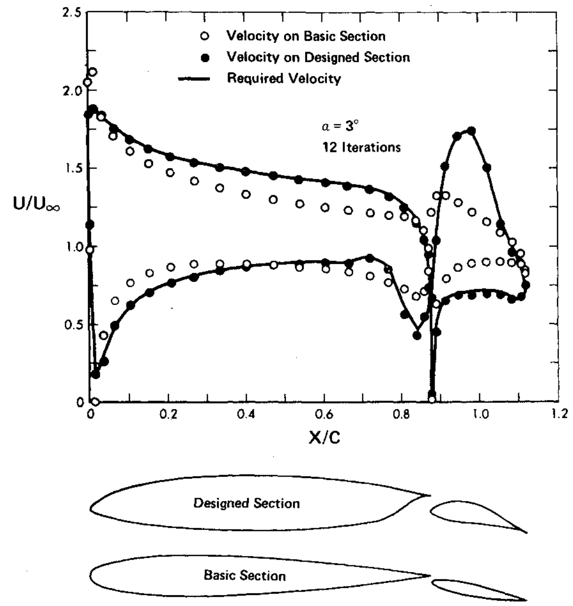


**Fig. 8 Two-component design.**

error. In the cases shown in Fig. 6 this occurs at the 8th iteration for the flat element method and at the 6th iteration for the smoothing method. The trends observed in Fig. 6 are typical of those observed in tests of the design method on many single component sections with angles of attack ranging from −8 deg to +10 deg and employing various numbers of elements on each section. Of these results the data presented in Fig. 6 are representative of the results obtained from difficult design test cases.

The results of the 8th iteration of this test case using the flat element method are presented in Fig. 4. The reasons that this is a difficult test case are apparent from the figure. There is a high velocity peak at the nose and a considerable amount of loading in the rear 30% of the designed section. The geometry of the section alters from a maximum thickness of 12% of chord to approximately 16% of chord. The change in the thickness distribution is even more radical as the tail is designed from a thick, symmetric wedge shape to a thin, cambered, almost cusped shape. Despite these difficulties, this design is considered to be quite adequate.

An example of a rather arbitrary design problem, based on a NACA 0012 section at 0 deg angle of attack, is shown in Fig. 7. The upper surface velocities are increased and the lower surface velocities decreased by an amount that varies linearly from leading edge to trailing edge. The results in this case after 8 iterations are exceptionally accurate. The required results are in fact those one would expect, according to the simple design rules of Abbott and Von Doenhoff,[7] had one distributed the NACA 0012 thickness distribution around a NACA mean line of $a=0$, at an angle of attack of 4.56 deg. The designed section is inclined at an angle of approximately 5 deg to the freestream and the thickness distribution has changed very little. The very accurate results which are achieved in this case are attributed mainly to the fact that there is so little change in the thickness distribution.

A two-component section was designed to test the performance of this design method on multicomponent airfoil sections. The required velocity distribution was obtained from a known two-component airfoil section at an angle of attack of 3 deg. The results of the design process after 12 iterations, when the convergence criterion was satisfied, are shown in Fig. 8. The basic section was composed of two NACA 0012 sections, each scaled to span the desired $x$ coordinates. The required surface velocities on both the main component and flap were supplied together with the values of the stream functions, $\psi_1$ and $\psi_2$, on the main component and flap, respectively. Sixty surface elements were used – forty on the main component and twenty on the flap – as the analysis method[6] has shown that this is adequate for an accurate solution.

In this example the starting flap section was set at a deflection of 10 deg and spaced down from the main section by 1% chord. The initial flap angle and spacing are not critical because these parameters are quickly adjusted by the design process. Since the $x$ coordinates are not altered during the design process, the flap chord will change if the final flap deflection is different from the starting deflection. Thus, if a particular final flap chord is desired, it will help to guess at the final deflection and set the starting flap at that angle.

The results show good agreement between the required and the designed velocity distributions. This is quite remarkable as, again, this case represents a difficult problem for the design method. Near the trailing edge of the main component there are quite drastic changes in thickness distribution and camber. On the flap the maximum thickness changes from 12% of chord to about 30% and the camber changes are considerable. The flat element method was used in this case and was quite capable of handling the severe changes in slope which occur at the rear of the main component. At the same time it must be admitted that this is the region in which the largest velocity errors occur on the main component. The largest velocity errors on the flap occur on the underside, due to the inability of the flap to develop its full thickness in 12 iterations.

The very practical problem of designing a flap where the main component is fixed demonstrates some of the important features of this design method. The velocities required of the main component are assumed to be unknown and are replaced by the values calculated in the analysis of the section designed in the previous iteration. The geometry and location of the main component are held constant regardless of results of the design process.

In choosing values of $\psi_1$ and $\psi_2$, the stream functions for the main section and the flap, it is the difference $\Delta\psi = \psi_2 - \psi_1$ that is important. $\psi_1$ can be taken as any arbitrary value. $\psi_2$ is taken as the fixed amount $\Delta\psi$ from the value of $\psi_1$ calculated from the previous analysis, that is, $\psi_2^{(r)} = \psi_1^{(d-1)} + \Delta\psi$. Since $\Delta\psi$ represents the volume flow through the slot, keeping $\Delta\psi$ constant maintains the correct slot width from one iteration to the next.

The value of $\psi_2 - \psi_1$ can be determined from the required slot width together with the estimated velocity through the

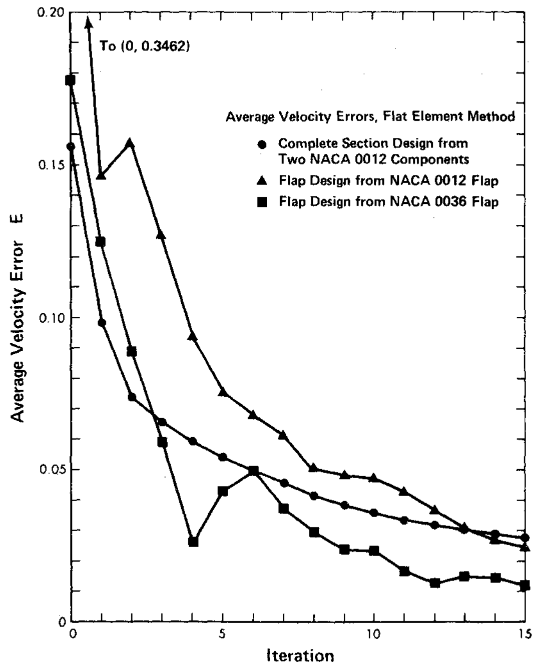


**Fig. 9 Convergence of two-component design.**

slot. Experimental investigations[8] have shown that the slot width should be about 3% of chord in order to avoid interference between the wake from the main component and the flap boundary layer.

Convergence toward the required solution for these two examples is shown in Fig. 9, where the average velocity error is plotted against iteration number. Convergence for the case of the flap design alone takes more iterations than for the complete design.

One reason for the slow convergence of the flap design of the large change in thickness required between the starting flap and the final design. The design method is able to alter camber and incidence quickly, but requires a larger number of iterations to change thickness. Figure 9 shows not only a much more rapid convergence if a NACA 0036 section is chosen for the basic flap, but also a more accurate final design.

In the complete two-component design starting from two NACA 0012 sections, the average error drops off more slowly than in the single component case seen in Fig. 6. Hence the convergence criterion of an improvement of less than 1% of the original error does not stop calculations until the 12th iteration. The slower rate of convergence and higher average velocity error are typical of two component section designs when compared with single component cases. This behavior is due to the general complexity of the geometry of multicomponent sections with highly curved sections near the trailing edges of forward components.

The average velocity errors for the two flap-design cases are based on the flap velocity distribution only. Since the main section geometry is being held fixed, any velocity distribution produced is assumed to be the correct one.

The amount of computer time used by this design method depends on both the number of elements used and the number of iterations performed. For a single component section using 40 surface elements and employing 8 or more iterations, the CPU time required is less than 0.34 seconds per iteration. This is for the complete program with all options shown in Fig. 3 using an Amdahl 470 V/6 computer. The analysis option takes considerable time, and when it is only performed on the final design the required time drops to less than 0.22 seconds per iteration. For two-component designs using 60 elements, the required time is less than 0.90 seconds per iteration. Using the analysis only on the final design reduces the required time to less than 0.45 seconds per iteration.

## Conclusions

A method of designing multicomponent airfoil sections in potential flow has been developed. This method is based on an accurate, efficient analysis method from which the design equations are derived. Each component of an arbitrary basic airfoil is modified in both thickness and camber to give a new section which has a required surface velocity distribution. This is done in an iterative manner which allows the designed geometry and the required velocity distribution to be altered at will at each iteration. It has been shown that this freedom results in a very powerful design tool which can be fitted to suit many of the constraints which occur in the design of modern airfoil sections.

Two techniques of determining the airfoil coordinates from the designed control points were considered. The flat element technique was found to result in less error and faster convergence than the smoothing method. This is because the latter method has a tendency to reduce the surface curvatures.

The design method provides suitably accurate solutions for both single and two-component sections. Generally the method will converge to an accurate solution in 8 iterations for a single component case and 12 iterations for a two-component case. The example of designing the flap alone shows the flexibility of this method. Although any arbitrary airfoil can be used as the basic section to start the design, faster convergence can be expected if the basic section has about the same thickness as the required final design, because changes to thickness take place more slowly than do camber changes in the design process.

The power of this method as a design tool lies in its flexibility. Because of this, many variations on this method are possible. The performance of each such variation will have to be examined individually.

## Acknowledgment

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